REFERENCES

1. M. PERLMUTTER and R. SIEGEL, Heat transfer by combined forced convection and thermal radiation in a heated tube, *Trans. Am. Soc. Mech. Engrs* (C), *J. Heat Transfer* **84**, 301–311 (1962).
2. R. SIEGEL and M. PERLMUTTER, Convective and radiant heat transfer for flow of a transparent gas in a tube with a gray wall. *Int. J. Heat Mass Transfer* **5**, 639–660 (1962).
3. R. SIEGEL and E. G. KESHOCK, Wall temperatures in a tube with forced convection, internal radiation exchange and axial wall heat conduction, NASA TN D-2116, March 1964.
4. J. C. CHEN, Laminar heat transfer in tube with nonlinear radiant heat-flux boundary condition, *Int. J. Heat Mass Transfer* **9**, 433–439.
5. J. SELLARS, M. TRIBUS, and J. KLEIN, Heat transfer to laminar flow in a round tube or flat conduit—the Graetz problem extended, *Trans. Am. Soc. Mech. Engrs* **78**, 441–448 (1956).
6. W. M. KAYS, *Convective Heat and Mass Transfer*, pp. 133–142. McGraw-Hill, New York (1966).

Int. J. Heat Mass Transfer. Vol. 12, pp. 1187–1191. Pergamon Press 1969. Printed in Great Britain

TEMPERATURE DISTRIBUTIONS FOR INSCRIBABLE NONCIRCULAR DUCTS HAVING CONSTANT WALL HEAT FLUX AND CONSTANT VELOCITY FLOW

JOSEPH T. PEARSON

School of Mechanical Engineering, Purdue University, Lafayette, Indiana, U.S.A.

(*Received* 26 *February* 1968 *and in revised form* 3 *February* 1969)

NOMENCLATURE

A,	cross-sectional area of duct;
A_i,	area subtended by the ith flat side of duct;
A_j,	area subtended by the jth curved side of duct;
B,	a constant;
e_i,	distance measured along the ith side of duct from a corner to the tangent point of the inscribed circle;
k,	thermal conductivity of fluid;
L_i,	width of the ith flat side of duct;
m,	total number of flat sides of an inscribable duct;
n,	total number of curved sides of an inscribable duct;
n,	inward normal to duct's surface;
P,	perimeter of duct's cross-section;
q_w,	wall heat flux at duct's inside surface;
r, θ, z,	cylindrical coordinates;
r_h,	hydraulic radius and radius of circle inscribed in duct's cross section, $r_h = 2A/P$;
$\overline{r^2}$,	square of radius of gyration of duct's cross-section;
$S(r, \theta) = 0$,	equation for duct's inside surface;
T,	temperature;
T_0,	temperature at duct's entrance;
t,	time;
u,	velocity of fluid in z direction;
v_r,	velocity of fluid in r direction;
v_θ,	velocity of fluid in θ direction;
x, y, z,	Cartesian coordinates;
Z,	thermal entrance length;
α,	thermal diffusivity of fluid.

Subscripts

b,	bulk temperature;
c,	center or axis of duct;
i,	the ith flat side;
j,	the jth curved side;
n,	normal to duct's surface;
0,	duct's entrance;
s,	at duct's surface;
1,	some point in fluid after thermal entrance region.

INTRODUCTION

FLUID flow passages in modern heat exchange systems are often of noncircular cross-sectional shape. In fact, heat transfer passages which are employed in aircraft, missiles, space vehicles and nuclear reactors are more often than not noncircular. In these engineering applications it is of utmost importance that the heat exchange systems operate as efficiently and as safely as possible. However, at the present time comprehensive design criteria which can be utilized to analyze the fluid mechanics and heat transfer associated with the various components of these systems are sparse. This is because it is quite difficult to simultaneously analyze the problem of combined fluid mechanics and heat transfer in irregularly shaped channels as has been pointed out by Sparrow and Haji-Sheikh [1] and Wilson and Medwell [2].

To ease this situation one often seeks to separate the heat transfer and fluid mechanics problems and thus reduce the governing system of equations to tractible forms and in turn obtain "approximate solutions". The technique most often used for separating the fluid mechanics from the heat transfer is to assume that the fluid properties are independent of temperature, while the technique most often used for separating the heat transfer from the fluid mechanics is to assume that the fluid velocity is constant across the duct's cross-section. The assumption of fluid properties independent of temperature is used often and its implications are in many cases not difficult to ascertain. On the other hand, the assumption of slug flow in most situations does not lead to heat transfer results which can be directly applied to the physical problem. However, there are techniques available, Hartnett and Irvine [3] and Claiborne [4], which transform slug flow heat-transfer solutions into solutions which approximate the physical situation.

ANALYSIS

Consider the heat transfer in an arbitrary inscribable duct which is illustrated in Fig. 1. The cross-sectional size and shape of the duct do not vary in the axial z direction. The cross-section of the duct is composed of m flat and n curved sides within which a circle can be inscribed. The flat sides are each tangent to the inscribed circle and the curved sides are each circular arcs which coincide with the inscribed circle. The axis of the duct is selected so that it coincides with the axis of the inscribed circular cylinder. This specification is by definition the most general form of an *inscribable duct*. It is of interest to note that the radius of the inscribed circle is the hydraulic radius of the duct which is defined as twice the cross-sectional area divided by the wetted perimeter. Many ducts of engineering importance are inscribable. Included are:

1. All triangular ducts.
2. All regular polygonal ducts.
3. All regular star shaped ducts.
4. The circular tube.

For steady incompressible slug flow (u = constant, $v_r = v_\theta = 0$), negligible thermal radiation, constant fluid properties and constant wall heat flux, the energy equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} = \frac{u}{\alpha}\frac{\partial T}{\partial z} \tag{1}$$

and the boundary conditions can be written as:

$$\frac{\partial T}{\partial n} = -\frac{q_w}{k} \text{ on } S(r,\theta) = 0, \tag{2}$$

where $S(r,\theta) = 0$ is the equation for the duct's inside surface and

$$T(r,\theta,0) = T_0(r,\theta), \tag{3a}$$

i.e. the temperature distribution is known at the entrance to the duct or

$$T(r_1,\theta_1,z_1) = T_1 \tag{3b}$$

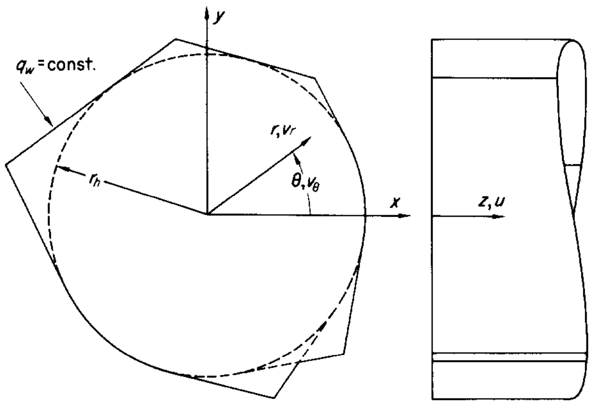


FIG. 1. Duct within which a circle can be inscribed. This is the general configuration of the *inscribable duct*.

where $z_1 > Z$, i.e. the temperature is known at some point in the fluid downstream of the entrance region.

It will now be shown that the solution of Equation (1) and its boundary conditions is

$$T(r,\theta,z) = T_c(z) + \frac{q_w}{2kr_h}r^2 \quad \text{for} \quad z > Z \tag{4}$$

where $T_c(z) \equiv T(0,\theta,z)$ is the temperature along the axis of the duct, Z is the thermal entrance length and r_h is the hydraulic radius.

It is noted that since $S(r,\theta)$ is independent of z

$$\int_S q_w \mathrm{d}S = \text{constant}. \tag{5}$$

This is the constant heat input per unit length of duct boundary condition. This combined with the condition that u = constant gives rise to a temperature field in the fully developed region of the duct which is a linear function of z. Therefore,

$$\frac{\partial^2 T}{\partial z^2} = 0 \quad \text{for} \quad z > Z. \tag{6}$$

Employing this result and using the proposed expression for T, equation (4), in equation (1) yields

$$\frac{\mathrm{d}T_c}{\mathrm{d}z} = \frac{2\alpha q_w}{kur_h} \quad \text{for} \quad z > Z. \tag{7}$$

Integration of this equation gives

$$T_c = \frac{2\alpha q_w}{kur_h} z + B \quad \text{for} \quad z > Z \tag{8}$$

where B is the constant of integration which can be evaluated from either boundary condition (3a) or (3b). Now the only other condition which equation (4) must meet is that of boundary condition (2) which can be rewritten as

$$\frac{\partial T}{\partial n} = \frac{\partial T}{\partial r}\frac{\mathrm{d}r}{\mathrm{d}n} + \frac{\partial T}{\partial \theta}\frac{\mathrm{d}\theta}{\mathrm{d}n} = -\frac{q_w}{k} \quad \text{on} \quad S \tag{9}$$

where dr and r_s dθ are the two orthogonal vector components of $-$dn as illustrated in Fig. 2. Employing equation (4) in equation (9) one obtains

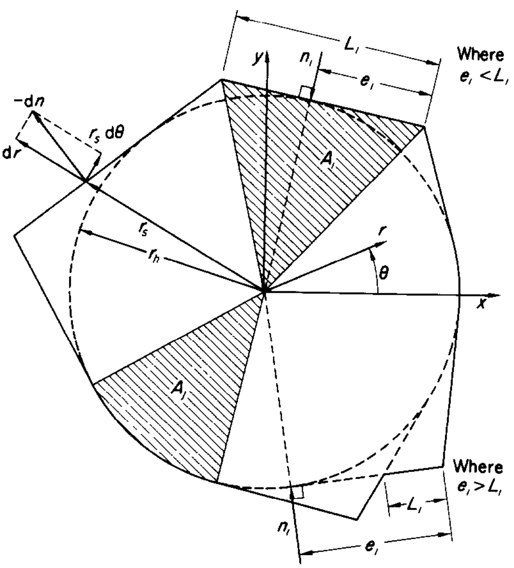


FIG. 2. Cross-section of the generalized inscribable duct.

$$1 = -\frac{r_s}{r_h}\frac{\mathrm{d}r}{\mathrm{d}n} \tag{10}$$

since $\partial T/\partial \theta$ is zero and dθ/dn is either finite or zero. Noting that dr is to r_h as $-$dn is to r_s, equation (10) can be rewritten as

$$1 = -\frac{r_s}{r_h}\left(-\frac{r_h}{r_s}\right) = 1. \tag{11}$$

Therefore the temperature field is indeed given by equation (4).

It should be mentioned that the problem being considered here is analogous to the problem of determining the temperature distribution in a long solid bar of inscribable cross-sectional shape when the surface heat flux is constant. With change of variable $t = z/u$, the transient temperatures and temperature distributions are given by the equations presented here for times greater than the time required for the shape of the temperature profile across the bar to become fully developed.

It is seen by examination of equations (4) and (8) that if the wall heat flux is constant, the isothermal surfaces are paraboloids whose axes coincide with the duct's axis. The temperature varies as the square of the distance from the duct's axis and linearly with distance down the duct. In the r, θ plane, isothermal lines are circles whose centers coincide with the center of the inscribed circle so that the inscribed circle is itself an isothermal line. If the fluid in the duct is being heated, the coldest region at any cross-section is at the center of the inscribed circle and the hottest region is in the duct's corner fartherest from the center of the inscribed circle.

T_c can be eliminated by integrating T over the cross section to obtain the bulk temperature.

$$T_b = \frac{1}{A}\int_A T\,\mathrm{d}A = T_c + \frac{q_w}{2kr_h}\overline{r^2} \tag{12}$$

where $\overline{r^2}$ is the square of the radius of gyration of the cross-section.

$$\overline{r^2} \equiv \frac{1}{A}\int_A r^2\,\mathrm{d}A = \text{constant}. \tag{13}$$

So, T_b is equal to T_c plus a constant in the fully developed region. Solving for T_c and substituting into equation (4) gives

$$T = T_b + \frac{q_w}{2kr_h}(r^2 - \overline{r^2}) \quad \text{for } z > Z. \tag{14}$$

Furthermore, T_b can also be eliminated by making an overall energy balance in the entrance region as well as the rest of the duct. This yields

$$T_b = T_{0b} + \frac{2\alpha q_w}{kur_h} z \tag{15}$$

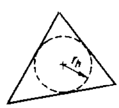
Table 1. *Summary of results*

Temperature distribution for $z > Z$

$$T = T_c + \frac{q_w}{2kr_h} r^2$$

$$T = T_b + \frac{q_w}{2kr_h}(r^2 - \overline{r^2})$$

$$T = T_{0b} + \frac{q_w}{2kr_h}\left(\frac{4\alpha z}{u} + r^2 - \overline{r^2}\right)$$

Cross-sectional shape of duct	Square of radius of gyration
 Inscribable	$\overline{r^2} = \frac{r_h^2}{2} + \frac{1}{6P}\sum_{i=1}^{m}[e_i^3 + (L_i - e_i)^3]$
 Regular star	$\overline{r^2} = \frac{r_h^2}{2} + \frac{1}{6L}[e^3 + (L - e)^3]$
 Regular polygon	$\overline{r^2} = \frac{r_h^2}{2}\left(1 + \frac{1}{3}\tan^2\frac{\pi}{m}\right)$
 Triangular	$\overline{r^2} = \frac{r_h^2}{3} + \frac{ab + bc + ca}{6} - \frac{4}{3}\frac{abc}{P}$

where T_{0b} is the bulk temperature at the entrance. If the fluid enters the duct at constant temperature, T_0, then $T_{0b} = T_0$.

By combining equations (8), (12) and (15) it is found that

$$B = T_{0b} - \frac{q_w}{2kr_h}\overline{r^2}. \tag{16}$$

Thus one can determine the temperature of the fluid at any location past the thermal entrance region by knowing only the temperature of the fluid at the duct's entrance. If on the other hand the temperature boundary condition is given by equation (3b) instead of (3a)

$$B = T_1 - \frac{q_w}{2kr_h}\left(r_1^2 + \frac{4\alpha z_1}{u}\right). \tag{17}$$

The square of the radius of gyration, $\overline{r^2}$, can be found by integrating over each region A_i and A_j as shown in Fig. 2. The results add to give

$$\overline{r^2} = \frac{r_h^2}{2} + \frac{1}{6P}\sum_{i=1}^{m}\left[e_i^3 + (L_i - e_i)^3\right] \tag{18}$$

for any inscribable duct. Also for any triangular duct

$$\overline{r^2} = \frac{r_h^2}{3} + \frac{ab + ac + bc}{6} - \frac{4}{3}\frac{abc}{P} \tag{19}$$

where a, b and c are the lengths of the three sides. And for any m sided regular polygonal duct

$$\overline{r^2} = \frac{r_h^2}{2}\left(1 + \frac{1}{3}\tan^2\frac{\pi}{m}\right). \tag{20}$$

A summary of these results is given in Table 1.

REFERENCES

1. E. M. SPARROW and A. HAJI-SHEIKH, Transient and steady heat conduction in arbitrary bodies with arbitrary boundary and initial conditions, *J. Heat Transfer* **90C**(1), 103–108 (1968).
2. N. W. WILSON and J. O. MEDWELL, An analysis of heat transfer for fully developed turbulent flow in concentric annuli, *J. Heat Transfer* **90C**(1), 43–50 (1968).
3. J. P. HARTNETT and T. F. IRVINE, Nusselt values for estimating turbulent liquid metal heat transfer in non-circular ducts, *A.I.Ch.E. Jl* 3, 313–317 (1957).
4. H. C. CLAIBORNE, Heat transfer in noncircular ducts, Oak Ridge National Laboratory Report 985 (1951).

Int. J. Heat Mass Transfer. Vol. 12, pp. 1191–1195. Pergamon Press 1969. Printed in Great Britain

RADIATION EFFECT ON THE ENTHALPY AND VELOCITY DISTRIBUTIONS OF A LAMINAR, COMPRESSIBLE, PLANAR FREE JET

M. M. ABU-ROMIA†

Polytechnic Institute of Brooklyn, Brooklyn, New York, U.S.A.

(*Received* 28 *August* 1968 *and in revised form* 3 *February* 1969)

NOMENCLATURE

- a_i, functions defined by equations (26);
- b_i, functions defined by equations (26);
- C_p, specific heat at constant pressure;
- F, velocity function;
- G, a constant defined by equation (10b);
- G_i, functions governed by equations (21);
- H, total heat released at the jet entrance;
- h, specific enthalpy;
- h^*, specific enthalpy of the reference state;
- K, thermal conductivity;
- K_p, Planck mean absorption coefficient;
- L, radiation loss parameter defined by equation (9);
- M, total momentum released at the jet entrance;
- m, a function defined by equation (15);
- n, radiation loss parameter defined by equation (9);
- P_r, Prandtl number;
- Q_r, radiation loss per unit mass;
- q_i, functions defined by equations (28);
- r_i, functions defined by equations (28);
- S, a similarity variable defined by equation (10a);
- T, temperature;
- U_o, axial velocity given by equation (15);
- u, v, velocity components in Cartesian system;
- x, y, spatial coordinates;
- Z_i, functions defined by equation (27).

Greek symbols

- Γ, a function defined by equation (18);
- η, a smilarity variable defined by equation (10a);
- μ, viscosity;
- ρ, density;
- ρ^*, density of the reference state;
- σ, Stefan-Boltzmann constant.

† Assistant Professor of Mechanical Engineering.